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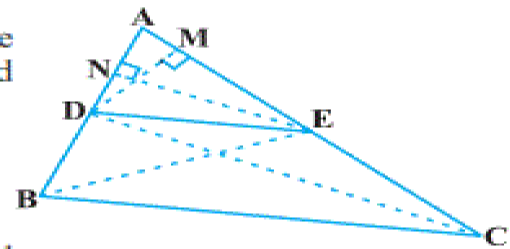
(Affiliated to CBSE Up to +2 Level)

CLASS: X | **SUBJECT: MATHEMATICS** | **DATE: 03-08-2021**

Theorem 6.1 : *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.*

Proof : We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.



To prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Now, area of Δ ADE (= $\frac{1}{2}$ base × height) = $\frac{1}{2}$ AD × EN.

Recall from Class IX, that area of Δ ADE is denoted as ar(ADE).

So, $\quad \text{ar(ADE)} = \frac{1}{2} \text{AD} \times \text{EN}$

Similarly, $\quad \text{ar(BDE)} = \frac{1}{2} \text{DB} \times \text{EN},$

$$\text{ar(ADE)} = \frac{1}{2} \text{AE} \times \text{DM} \quad \text{and} \quad \text{ar(DEC)} = \frac{1}{2} \text{EC} \times \text{DM}.$$

Therefore, $$\frac{\text{ar(ADE)}}{\text{ar(BDE)}} = \frac{\frac{1}{2} \text{AD} \times \text{EN}}{\frac{1}{2} \text{DB} \times \text{EN}} = \frac{\text{AD}}{\text{DB}} \qquad (1)$$

and $$\frac{\text{ar(ADE)}}{\text{ar(DEC)}} = \frac{\frac{1}{2} \text{AE} \times \text{DM}}{\frac{1}{2} \text{EC} \times \text{DM}} = \frac{\text{AE}}{\text{EC}} \qquad (2)$$

Note that Δ BDE and DEC are on the same base DE and between the same parallels BC and DE.

So, $\quad \text{ar(BDE)} = \text{ar(DEC)} \qquad (3)$

Therefore, from (1), (2) and (3), we have :

$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}} \qquad \blacksquare$$